Review and Maintenance Programme (RAMP) Mathematics and Statistics

Themes in the research literature

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1. Introduction

In the second half of 2014 the Ministry of Education initiated a process to review all materials funded and managed by them to support learning in the senior secondary school years. These are the years of schooling when achievement is predominantly assessed by achievement standards that build towards NCEA qualifications (the National Certificate of Educational Achievement). The process was given the acronym RAMP (Review and Maintenance Programme), and the stated focus was to ensure "that materials that support NCEA continue to be aligned with *NZC*[1] and support the development and use of quality teaching and learning programmes in the secondary school".[2] For the purposes of the review, these support materials were to include all those associated with NCEA: the achievement standards themselves; the matrix of key outcomes that accompanies the suite of achievement standards at each of NCEA Levels 1–3; student exemplars and other assessment resources; and any specified conditions of assessment. The online senior subject guides were also within the review's scope.

The Ministry of Education has sought several types of external input into the review process. An advisory group with demonstrated curriculum leadership and pedagogical expertise in the relevant learning area has the role of providing 'on the ground' expertise related to the challenges of teaching and learning. The Ministry has also requested a literature search for recent research related to the implementation of NCEA in the relevant learning area and/or the uptake and enactment of *NZC* in the final three years of schooling, informed by wider research of achievement in New Zealand across the years of schooling and by any associated policy debates. External input from teacher and student focus groups was planned for a later stage of the review process. Achievement data for at least the three previous years, along with feedback from relevant groups in the New Zealand Qualifications Authority (NZQA) and the Ministry itself, provided internal sources of feedback. These processes were implemented concurrently for the Mathematics & Statistics and Science learning areas of *NZC*.

This report provides input to the RAMP process from the literature review of mathematics and statistics education in the New Zealand senior secondary school context. The review took place across a relatively short time span in late 2014.[3] Three specific areas of importance to the Ministry of Education were outlined for the advisory group and the literature review team as:

- the critical connection between *NZC*, teaching and learning, and NCEA
- the needs of priority learners

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[1] *The New Zealand Curriculum (NZC*), which provides an overarching national curriculum structure for all the years of schooling (Years 1–13).

[2] Ministry of Education briefing materials.

[3] An annotated Endnote file constituted the second source of input from the literature search.

- the effect of support materials on school programmes.

With these priorities in mind, we, the NZCER reviewers, searched for all the relevant local literature we could locate. A small number of large-scale international reviews were used to help organise the mainly smaller local studies and to check for emergent issues that might be of interest to the Ministry as they determine their next steps in the provision of curriculum support for senior mathematics and statistics learning. Search and selection processes are described in the Appendix. We were surprised by the number of New Zealand-based papers we found. One hundred references were added to the Endnote file before we stopped, confident that we had captured the main themes.

The following sections summarise the key findings, organised to reflect the areas of concern for the RAMP process and informed (where appropriate) by our awareness of current concerns being debated in the international mathematics and statistics education communities. Note that this report does not seek to reach specific conclusions, which we see as the prerogative of the Ministry's internal RAMP team. However, the manner in which we have structured the results of the literature review process inevitably represents our thinking about the significance of the papers we found.

It is important to note that one of us (Alex) is a prominent member of the statistics education community in New Zealand, as well as being involved with the wider mathematics community. The other member of the review team (Rose) was closely involved in the RAMP science review and has a broad general research background in exploring NCEA and generic issues of *NZC* implementation, with a specific focus on the role of key competencies in transforming learning. We have used our diverse backgrounds to reflect critically together on the structuring of the key themes in this review.

A note on terminology

In this document we follow a consistent style for *NZC* terminology:

- "Mathematics & Statistics"—refers to a learning area within *NZC*
- "Mathematics" or "Statistics"—refers to the two strands within the Mathematics & Statistics learning area.
- "mathematics and statistics"—refer to the separate disciplines, independently of their inclusion in NZC.

A brief reflection on framing

There is a clear divide in the Mathematics & Statistics learning area of *NZC* between the manner in which the Mathematics strand is developed and the manner in which the Statistics strand is developed. This divide creates a number of tensions. These were apparent in some of the local literature we reviewed, and Alex has experienced them first hand through his active involvement in the New Zealand Statistical Association and in research across the whole spectrum of the learning area. We begin the review by discussing the nature of the divide between the two strands, how and why it has evolved, and the nature of the resulting and ongoing tensions. This discussion is presented at the very beginning of the review because the tensions identified have an impact on much that follows in the subsequent sections.

The first of the RAMP review series identified a paradigm shift in science education. This shift in how issues are framed and understood impacts on the way in which the implications of research findings might be used to inform future actions (Hipkins & Joyce, 2015). It will become clear that this tension within the wider Mathematics & Statistics learning area also plays out in how issues within the domain are understood, and therefore has an effect on possibilities for future action. Alerted to the impact of this tension by the recently completed science review, we have endeavoured from the outset to be more explicit in our discussion of its dynamics in this second review for the RAMP series.

Issues and challenges we addressed in separate sections of the science review are more tightly interwoven in this review. There are fewer, but slightly lengthier, sections. Where findings from the science review are also applicable to Mathematics & Statistics, we have mostly chosen to paraphrase and cross-reference from that review, thus keeping the focus of this report on substantive new issues.

2. Mathematics and Statistics in *NZC*

This section contrasts the alignment between the intent of the curriculum and its enactment in Mathematics and in Statistics. We present evidence that the alignment between the high-level intent of *NZC* and the intended learning is quite different in the two major strands of the learning area, and we explore a range of challenges and issues related to these differences.

A brief discussion of the structure of the learning area

NZC states that the disciplines of mathematics and statistics “are related but different ways of thinking and solving problems” (Ministry of Education, 2007, p. 26). Reflecting these differences, their achievement objectives are developed as separate strands within the overall learning area.

For *NZC* Levels 1–6 there are two strands for Mathematics (Number and Algebra; Geometry and Measurement) and one for Statistics. Learning for younger students places a comparatively stronger emphasis on numeracy and the use of mathematics in context. Level 6 presents something of a transition zone before the structural changes at Levels 7 and 8, when the learning area splits into just two strands: Mathematics and Statistics. At Levels 7 and 8:

- Mathematics is largely concerned with algebra (*Patterns and relationships*, and *Equations and expressions*) and *Calculus*—the emphasis is on formal mathematical reasoning, with a de-emphasis on application to real-world contexts (i.e., it is ‘academic’ in the traditional sense of the learning area).
- Statistics retains the three sub-headings of the previous levels (*Statistical investigations*, *Statistical literacy* and *Probability*). The strong link to real-world contexts remains integral to learning, at least in theory.

Why learn Mathematics & Statistics?

The RAMP review for the Science learning area documented considerable policy activity and debate about why all students should have rich and engaging opportunities to learn science at school (Hipkins & Joyce, 2015). We did not find an equivalent collection of policy papers for the Mathematics & Statistics learning area. This is interesting because the central issues—how the learning should enrich students’ intellectual development, keep their learning pathways open, and provide them with important and complex capabilities they will need in their lives beyond school—are just as important and salient in the Mathematics & Statistics learning area.

Conversely, the science literature we found did not present overt debate about the advisability of structuring the Science learning area into traditional disciplines (biology, chemistry, physics, and the recent addition of earth sciences). By contrast, discussion of the structure of the Mathematics & Statistics learning area is alive and well, particularly in the Statistics strand. This is covered in more depth in the sub-section below headed “The paradigm shift in statistics education”. It seems possible that the overall policy debate about purposes for learning in Mathematics & Statistics has been diffused by the structural compromise reflected by the actual and conceptual divisions between mathematics and statistics.

The high-level learning area statement in the front part of *NZC* (Ministry of Education, 2007) says that Mathematics & Statistics entails the exploration and use of patterns and relationships in quantities, space, and data. It equips students “with effective means for investigating, interpreting, explaining, and making sense of the world in which they live” (p. 26). When this learning area statement is read in combination with other transformative signals from the front part of *NZC* (vision, values, key competencies, etc.), there is a sense that curriculum development work is needed to show students the relevance of mathematics to their lives beyond school.

Reflecting this emphasis, statistical education in New Zealand schools has been moving away from the accumulation of facts and procedures and towards a conceptual understanding of statistical thinking (see, for example, Pfannkuch, Arnold, & Wild, 2011). The structure of the Statistics strand reflects this shift. The achievement objectives are framed within the statistical enquiry cycle, the evaluation of statistically based reports, and in investigations of chance, with an emphasis on supporting students to develop their statistical literacy.

However, compared with this more contemporary approach developed through the Statistics achievement objectives, the achievement objectives remain more traditional in Mathematics. These continue to identify abstract mathematical knowledge and skills, without reference to the setting of this learning. The NCEA achievement standards are more closely aligned with the traditional curriculum signals from the back half of *NZC* than with the transformative intent of the front half, particularly in Mathematics, and to a lesser extent in Statistics.

Pfannkuch and Wild (2013) and Forbes (2014) detail how the education committee of the New Zealand Statistical Association (NZSA) has helped to ensure that the Statistics achievement standards do evolve in ways that reflect the curriculum. This group has worked closely with both the Ministry of Education and NZQA on revisions of the achievement standards, and on the assessment exemplars that support the interpretation of each standard. This has been an especially powerful partnership, given that Maxine Pfannkuch led the writing of the Statistics strand of *NZC*.

The development of thinking in mathematics and statistics

Among the reasons given for learning mathematics, *NZC* highlights thinking skills, thereby aligning the learning area with the key competency *thinking*. *NZC* states that thinking skills are strengthened through problem solving, creating models, predicting outcomes, conjecturing, and seeking patterns and generalisations (p. 26). All these types of learning opportunities can be applied to real-world issues and challenges. They can equally be applied to thinking within the purely mathematical sphere, which can build strategies for developing logical, structured and rigorous thinking. Thus the tension between traditional and more contemporary approaches to mathematics and statistics learning is not resolved by appealing to the development of robust logical thinking skills as a main purpose for learning mathematics and statistics. If anything, *both* applied and abstract thinking are seen to be necessary and valuable. However, as subsequent discussion in this review will show, this both/and compromise perpetuates tensions between traditional and contemporary approaches to teaching mathematics rather than helping to solve them.

NZC and the Number strand

Some of the tensions already mentioned are reflected in the contested nature of the relationship between a 'number-based' approach to teaching and learning mathematics and the development of thinking in the other strands of Mathematics & Statistics. For example, we found research commentary with implications for thinking about the relationship between the Number Framework[4] that supports *NZC* and the Geometry & Measurement and Statistics strands of the learning area. Many of the concepts in these areas are not intrinsically number concepts; examples are spatial reasoning and the idea of statistical distribution.

Evidence from the literature

- There is evidence that enforcing a number-based approach in statistics can damage the development of reasoning in non-number concepts. For example, stressing the multiplicative way of computing the mean (or standard deviation) can hinder conceptual understanding of the key statistical concepts of centrality and variation (Cobb, 2007). The implication is that placing statistical and geometric thinking within the Number Framework is problematic.
- Cobb further pointed out that in a technological era, statisticians can access methods such as bootstrapping/resampling that are more statistically robust than traditional computational methods. Calculable assumptions of 'normality' (mean, median, mode, etc.) belong to an era when limits were put on statistical methods by computational constraints (the 'tyranny of the computable'). Perpetuating the emphasis on these number concepts in statistics puts secondary statistical ideas before the primary ones that now prevail (Cobb, 2007).

The paradigm shift in statistical education

The paradigm shift in statistical education, reflected in Cobb's commentary above, actually began with the 1992 curriculum (Ministry of Education, 1992). This was already acknowledged as being

[4] http://nzmaths.co.nz/new-zealand-number-framework

at the cutting edge of curriculum development. Begg and Pfannkuch (2004, p. 12) quote Jane Watson, a leading international statistics educator, who requested that the Statistics learning area in *NZC* curriculum should "not remove the emphasis that is in the 1992 curriculum. It is a world leader." Begg et al.'s literature review, undertaken to inform the revision of the 1992 Statistics strand in the 2007 *NZC*, is still very helpful, not just for the extensive references but for the range of clear recommendations. These included:

- retaining *thinking* as a major focus of a 21st century curriculum for both mathematics and statistics
- developing ideas about what constitutes mathematical thinking and what constitutes statistical thinking
- including key statistical concepts, such as understanding variation and reasoning with uncertainty and with probability
- the use of investigations and problem-solving approaches
- an explicit focus on the communication of mathematical ideas
- foregrounding statistical literacy as the purpose for learning in this strand (thereby closely aligning Statistics with the curriculum aims)
- ensuring contemporary inquiry approaches using technology are reflected in development of learning
- developing clear links between statistics and probability
- developing coherence between statistics and mathematics.

These recommendations were largely incorporated into the Statistics strand of the learning area.[5] There is a strong focus on data-driven investigations, centred around the statistical inquiry cycle introduced by Wild and Pfannkuch (1999). The sub-strand called *Investigating Statistical Reports* in the 1992 curriculum was replaced by a *Statistical literacy* sub-strand in *NZC*. The latter involves interpreting statistical information and evaluating data-based arguments that are made by other people.

We found evidence of considerable research activity to support teachers to develop the *NZC* emphasis on statistical reasoning using a range of innovative pedagogical approaches, many of which exploit the potential in e-learning tools. Much of this activity has been generated by one extended research team.

Evidence from the literature

- Hands-on simulations build younger secondary students' understanding of statistical concepts, such as recognising and describing sampling variability and using a sample to make inferences about a population. The authors argue that teachers need to think about how to immerse their students in a coherent set of statistical experiences that support them to shape and discuss inferential ideas. Such learning experiences lead on to more formal inferential

[5] The exception is development of coherence between mathematics and statistics,

reasoning at the senior secondary level. This approach runs counter to the traditional approaches to teaching statistics, which emphasise computation rather than reasoning (Arnold, Pfannkuch, Wild, Regan, & Budgett, 2011).

- IT can be used to stage the development of big ideas in statistical inference and make them more accessible to secondary students, as well as to adult education and to introductory courses in the tertiary sector. IT tools can support learners to make visual comparisons of the salient features of data graphs, supporting them to develop key ideas in statistical inference, building from a less formal to an increasingly formal approach to statistical reasoning (Wild, Pfannkuch, & Regan, 2011).
- The visual affordances yielded by computer simulations of graphical representations of data can be used to successfully support the development of specific target concepts such as sampling variation. Students who do not have strong computational skills, as well as more mathematically able students, can strengthen their informal inferential reasoning capabilities when these approaches are used (Pfannkuch et al., 2011).
- One team of researchers have developed a framework for assessing statistical literacy. The framework encompasses aspects such as critiquing the validity and limitations of good statistical information, and identifying misleading claims (Sharma, Doyle, Shandil, & Talakai'atu, 2012).
- Graphical representations of data—both where it is centred and how it is spread—link the concepts of variation and statistical distribution. Here ‘shape’ is used as a synonym for distribution, emphasising its geometric nature and its visual appearance (Watson, 2014).
- The development of the *NZC* Statistics strand has also been deliberately aligned to certain key practices of applied statistics in real-world 21st century contexts. A 2-year-long Teaching and Learning Research Initiative (TLRI) project developed dynamic visualisations of the process of bootstrapping, which is a key IT tool widely used by professional statisticians. The visualisations were shown to enhance conceptual understanding and hence successfully transform methods of learning about statistical inference in the senior secondary school (Pfannkuch, Forbes, Harraway, Budgett, & Wild, 2013).

The development of the *NZC* Statistics strand is still a work in progress. However, the achievement objectives for this part of the learning area align more closely with the high-level signals given by the front part of *NZC* than do the achievement objectives for the Mathematics strands. As outlined above, the learning emphasis is placed on the development of statistical reasoning to build capabilities in statistical literacy rather than on the development of ‘mathematical statistics’, where the learning emphasis is on the accumulation of facts and procedures, based on traditional computation. The rationale for this paradigm shift is that most people need statistical literacy in their lives, but only a very few need to develop their abilities in traditional statistical practices.

The transformation in statistical education has been strongly supported by the New Zealand statistical education community, which for some considerable time has been working on incremental curriculum changes that support the current philosophy—upwards of 30 years (Forbes, 2014; Pfannkuch & Wild, 2013). These authors describe a continuum of evolutionary changes rather than a single quantum leap. Hipkins (2014) recently interviewed a range of participants in this research programme, and she identified NZSA support as a key influence in achieving the shift in statistics education outlined above.

Mathematics in New Zealand schools

Our literature search did not uncover an analogous move in the Mathematics strand of senior secondary mathematics. It seems, at least anecdotally, that a more traditional approach to subjects such as algebra and calculus is typical. This was borne out when the then president of the New Zealand Mathematics Society, Dr Graham Weir, attended the New Zealand Statistical Association Education Committee meeting in 2013 to discuss the influences that had led to transformations in statistics education and how some of those lessons may apply to the mathematics fraternity.[6]

This situation would seem to reflect a lack of clear signals about *system-level* shifts in curriculum thinking about the Mathematics strand of *NZC*. However, notwithstanding the traditional emphasis retained by the achievement objectives of the Mathematics strand, we did find several smaller-scale initiatives that investigated and supported innovation at a local level, as the following examples show.

Evidence from the literature

- One study explored ways to integrate introductory algebraic thinking into the wider mathematics programme. The teaching approaches that were developed helped students in Years 9 and 10 to develop a conceptual understanding of algebra. This work also produced a diagnostic interview to help ascertain students' current algebraic reasoning skills (Linsell, Tozer, & Anakin, 2012).
- A 2005–2007 pilot programme in 22 schools explored the use of Computer Algebraic System (CAS) calculators to teach algebra and geometry in junior secondary mathematics. An evaluation of this pilot project reported significant improvements in student attitudes and motivation, partly attributed to the affordances of technology. The high-quality professional development that accompanied the project emphasised a less traditional pedagogical approach to mathematics, and this contributed to student engagement (Neill & Maguire, 2008).

[6] Alex Neill, personal attendance at the meeting

Mathematical modelling or problem solving as a way forward

Creating mathematical models and using practical applications are both mentioned in *NZC*'s high-level rationale for the Mathematics & Statistics learning area, as discussed at the start of this section. Mathematical modelling has the potential to situate mathematical thinking in real-life contexts and hence offers one possibility for resolving the current tension between the more contemporary approaches to statistics and the more traditional approaches to mathematics. The inclusion of these modelling approaches in teaching and learning, as well as in assessment, would more closely align mathematics with the high-level intent of *NZC*. We found both New Zealand and international research that could be helpful for supporting the development of modelling approaches in mathematics.

Evidence from the literature

- The use of open-ended modelling was identified as a productive pedagogical strategy in a recent Best Evidence Synthesis (BES) (Anthony & Walshaw, 2007). A subsequent summary of the BES states that "Open-ended and modelling tasks, in particular, require students to *interpret a context and then to make sense of the embedded mathematics*" (Anthony & Walshaw, 2009, pp. 13–14, emphasis added). This is an important endorsement given the rigour of the BES methodology. The emphasised phrase draws attention to the potential to develop strong contextual links.
- One design research project concluded that mathematical model-eliciting activities (MEAs) can elicit a diverse range of mathematical approaches from Year 11 students, and these can be a springboard for further mathematical conceptual development. Blueprints for the MEAs used in the research specify the principles and strategies that can be used to design follow-up activities (Yoon, Patel, Radonich, & Sullivan, 2011).
- A thesis explored mathematical modelling in the upper secondary school in Sweden and the use of mathematical modelling in the workplace (Frejd, 2014). Frejd stressed the relevance of mathematical modelling by students, including its formative potential, its utility, and its relevance to the practice of mathematics.
- In an earlier literature review Frejd also explored modes of assessment of mathematical modelling. He found that written tests tended to elicit an atomistic view of modelling competencies, whereas projects assessed more holistic modelling competence. He stressed the need for an elaborated view of the meaning of the quality of mathematical models in order to assess the quality of students' work (Frejd, 2013).
- Mathematical modelling embodies the same sorts of values and aesthetics as empiricism in science (examples given in this paper are precision, coherence, measurability and parsimony). Thus this aspect of mathematics has the potential to create more coherent synergies for students between mathematics and science curricula in schools (Tytler et al., 2008).

Problem solving has similarities to mathematical modelling as a pedagogical approach to the teaching of mathematics and statistics, and the two are often discussed in tandem. Whereas mathematical modelling involves building a mathematical system to describe a situation, problem solving is the process by which strategies are developed to obtain an answer to the problem.[7]

NZC refers to problem solving as the "practical application" of mathematics in everyday life, but this term can also refer to solving new or novel mathematical problems (i.e., more traditional abstract mathematical reasoning). However, problem solving that does not go beyond the routine application of mathematics to everyday contexts can lend a certain superficial saliency to the intended learning.

Evidence from the literature

- Ideally the intended mathematical learning and the problem students are trying to solve should be woven together as one complex activity (Zawojewski, Magiera, & Lesh, 2013). These authors propose a problem-driven mathematics curriculum with assessment located in the classroom. They suggest that richly contextualised problem-solving tasks can help assess the relationship between modelling, conceptual, and procedural knowledge. Like Yoon et al. (2011), they also mention the use of MEAs.

Both mathematics and statistics can involve student-led inquiry processes. Like other types of problem-solving activities, student-led inquiries could investigate real-life situations or problems, or have a focus on solving novel, context-free mathematical problems that students have not previously met. The place and importance of an inquiry-based pedagogy was also addressed in the literature we found.

Evidence from the literature

- Watson and Barton (2011) argue that mathematical modes of inquiry should be integral to mathematics and mathematics teaching, and hence also have a place in assessment. They say that teachers' fluency with mathematical modes of enquiry allows them to make a unique contribution to the learning of mathematics because it is something that a textbook or annotated website cannot provide.
- Halton (2009) argues similarly that solving novel mathematical problems or puzzles develops flexibility of thinking, helps students master a range of problem-solving strategies, and develops perseverance. He also argues that problem solving addresses the need of students to enjoy and engage with the intellectual challenge of mathematics.
- Inquiry-based classrooms can be effective in lifting the achievement of Pasifika students. One study found that a group of primary students constructed views of themselves as competent learners, both individually and collectively, when they took part in a collaborative classroom

[7] See http://nzmaths.co.nz/what-problem-solving for more.

culture that engaged them mathematically. Students demonstrated increased levels of engagement, and a more positive attitudes towards mathematics (Hunter & Anthony, 2011).

These various ideas come together in what the BES calls “worthwhile mathematical tasks” (Anthony & Walshaw, 2009). Such tasks send powerful messages to students about what mathematics is. The BES states that

> Effective teachers set tasks that require students to make and test conjectures, pose problems, look for patterns, and explore alternative solution paths. Open-ended and modelling tasks, in particular, require students to interpret a context and then to make sense of the embedded mathematics. (Anthony & Walshaw, 2009, pp. 13–14)

Opportunities or challenges?

Finally in this section we address three areas of debate related to wider support and pedagogy for teaching and learning mathematics and statistics. Each of these areas can be seen as either rich with potential or as highly problematic, depending on how in-principle curriculum and pedagogical decisions and actions are framed.

Use of contexts to support learning

The choice of contexts, and the reasons for using contexts, is one area where tensions between mathematics and statistics come sharply into focus. The high-level *NZC* discussion of the learning contribution made by Mathematics and Statistics notes that it is important to incorporate the use of real-life situations as well as hypothetical situations “drawn from a wide range of social, cultural, scientific, technological, health, environmental, and economic contexts” (Ministry of Education, 2007, p. 26). However, there is a need to distinguish between authentic, real-life contexts where the mathematics is used as an integral tool in achieving some *wider purpose*, and contexts that are there principally to serve *mathematical* teaching needs.

The latter could be thought of as ‘pseudo-contexts’. There is a place in mathematics education for them. As the BES notes, a tangible context, even if not necessarily *integral* to the learning intent, can be motivating when it provides “mathematics learning experiences that enable students to build on their existing proficiencies, interests, and experiences” (Anthony & Walshaw, 2009, p. 11). Contrived contexts can also help address the challenge that complexities of the context *per se* can carry the unintended risk of making the actual mathematics unduly onerous.

We found a number of papers that addressed the importance of real-life contexts in the teaching of mathematics and statistics.

Evidence from the literature

- In statistical investigations and in statistical literacy, information exists only in the context of the particular study. Therefore investigations in statistics need to be set in real-life contexts, and students need to be able to integrate statistical and contextual information (Wild & Pfannkuch, 1999).
- There is anecdotal evidence[8] that some students undertaking NCEA internal assessments in statistics are putting too much of their effort and output into the contextual aspects while ignoring the statistical aspects of their chosen study. The converse of this is also undesirable. Teachers need to have an idea of the appropriate balance between contexts and concepts.
- One mathematics teacher has used the context of road safety to produce two rich teaching and learning resources for the New Zealand Transport Agency (NZTA). One resource provides a purpose-built data-base for Year 10/11 students to investigate conditions that affect stopping distances while driving. A second resource develops computational skills in trigonometry while simultaneously investigating the time it takes to cross the centre line when distracted, at different speeds and angles of drift. Both these units of work aim to develop a specific aspect of mathematics and safe driving behaviour as a wider social outcome (New Zealand Transport Agency, 2014). Links between this teaching and learning innovation and the key competencies were recently elaborated in another Ministry of Education-funded resource.[9]
- A recent conference paper discussed the design principles that underpin these NZTA resources. The authors noted that the use of road safety as a context for learning mathematics had been highly engaging for students, and has had a positive impact on their driving attitudes and behaviours because the results of the inquiries created cognitive dissonance between what students thought they already knew and what they actually found to be the case in reality (Chamberlain & Hook, 2012).
- One study described the use of real-life contexts in one New Zealand Year 11 algebra lesson. Analysis of the lesson revealed that its success rested on the ways in which the learning tasks and their contexts were introduced, ongoing referral to the contexts, the consolidation of prior mathematics learning, and teacher questioning. The implication is that the teacher needs strong pedagogical skills to do these things well (Harvey & Averill, 2012).
- These researchers also quoted other researchers' concerns about assessment items framed by specific contextual considerations. Real-life problems are often more complex than the intent of an assessment question. Thus framing assessment items in context has the potential to penalise those students who try to take a broader set of realistic, generalised considerations about the context into account. To avoid this risk, Harvey and Averill argue that "word problems used in teaching and assessment [including NCEA] should include emphasis on

[8] This concern was expressed by both teachers and moderators at an NZSA education committee meeting attended by Alex Neill.
[9]http://nzcurriculum.tki.org.nz/Key-competencies/Key-competencies-and-effective-pedagogy/Engaging-examples-of-practice/Road-safety

realistic considerations in general and not just on very specific mathematical considerations" (Harvey & Averill, 2012, p. 44).

- Mathematics teachers participating in the Sport in Education project have been experimenting with using data (such as sports performance data from school sports or PE activities) as a context for learning aspects of measurement or statistics. The teachers' own inquiries show that such learning is highly engaging for students and can lift achievement levels for under-achieving students. However, contextualising takes time, and developing activities that use authentic contexts poses design challenges when meaningful use of authentic data, or another such real-life context, raises complex mathematical challenges that are beyond students' current mathematical abilities (Boyd & Hipkins, 2014).

Building coherence with other learning areas

Curriculum coherence is one of eight *NZC* principles that provide a foundation for schools' curriculum decision making. We found more papers than we had anticipated that addressed this challenge in the Science learning area (Hipkins & Joyce, 2015). It will be evident that some of this literature is also relevant to the current review, because much science inquiry set in authentic contexts also includes mathematical or statistical dimensions.

Evidence from the literature

- One study analysed Year 13 biology students' scientific reports for their biology assessment to evaluate their levels of statistical thinking. Students with higher levels of achievement in biology investigation tended to employ higher levels of statistical thinking. Misconceptions about aspects of statistical thinking were revealed. The teacher-researcher suggested that using sophisticated statistical analysis tools might result in students taking a black box approach to analysis, which hinders their statistical thinking. She suggested that teachers should actively facilitate the transfer of statistical thinking skills, and that collaboration between biology and statistics teachers could enable this (Jowsey, 2007).
- In a plenary talk at the 9th International Conference on Teaching Statistics (ICOTS) entitled "Curriculum Expectations for Teaching Science and Statistics", Jane Watson (2014) noted that statistics does not exist without context, and that science is a rich source of contexts for statistical investigations in mathematics. She concluded that "there is convincing evidence that science teachers have a model for inquiry that mirrors that of a statistical investigation" (p. 5).
- Another plenary talk at the same conference mapped the progress of statistical education over the last 60 years, moving forward to address new areas such as big data (Usiskin, 2014). Usiskin positioned statistics as an application of mathematics, but also noted that there are aspects of statistics that are not mathematical. Consequently, he noted, statistics needs to be taught as a subject in its own right, but the relationship between statistics and other

curriculum areas also needs to be developed, as does the relationship between mathematics and statistics.

A different type of case for integration relates to the building of financial literacy, or “financial capability”, as it is referred to in *NZC*, where it is introduced as a cross-curriculum construct (p. 39). The challenge of building students’ financial capability spans various learning areas, including Mathematics & Statistics. However, there is a mismatch between the high community demand for improved financial capability and the low emphasis on it in schools. A study of 22 secondary schools concluded that although both teachers and students value financial capability, it is not perceived to have the same status as the curriculum learning areas, or the key competencies, even though it is located in *NZC*. Teachers commented that a perceived lack of status for financial literacy unit standards is related to the lack of status given to financial literacy itself, and the consequent lack of prioritisation and resourcing. While financial literacy is partially assessed through unit standards, these standards and their associated assessments are subject to schools’ willingness to provide them and, in some cases, students’ selection of them (Neill, Berg, & Stevens, 2014).

Broadly speaking, adding the term ‘literacy’ to an area of interest could be seen as signalling a need to integrate learning from different areas of the curriculum. Examples in addition to financial literacy include health, scientific, statistical, digital literacies, and so on. One way or another, each of these quite different areas of capabilities includes a mathematical component, most commonly number and/or statistics. As the financial literacy study just cited shows, the current curriculum and assessment structures appear to act as a disincentive to more than a few pioneering teachers taking up these rich opportunities for integration. Again, this is a future-focused challenge that the Ministry of Education may wish to consider as part of the review.

Partnerships between schools and communities of interest

Finally in this section we address the challenge of establishing partnerships between schools and the wider community. As outlined above, *NZC*’s high-level signals are that students’ mathematics and statistics learning should be situated in real contexts of genuine relevance to them and their communities, and that students should be connected with community partners and supported to achieve and share the real outcomes of their learning work. Clearly, strong school–community connections are necessary for this aspiration to be realised in practice.

However, making strong school–community connections has implications for the way the curriculum is planned and organised; how mathematics and statistics learning interconnects with other learning; and the kinds of relationships, time frames and resources needed to support this interconnection, including resources provided by the mathematics and statistics communities, as well as both the local and the wider community.

Evidence from the literature

- Communities of professional mathematicians and statisticians can help provide strategic leadership to support knowledge development, and the sharing and coordination of school–mathematics and statistics community engagement initiatives. One recent case study showed how their proactive involvement can help strengthen networks of teachers and people working in intermediary roles across existing school–mathematics community engagement initiatives (Hipkins, 2014).
- The conference paper on NZTA's road safety resources noted that the richly contextualised learning created links between school and home, drawing parents and others in the community into conversations about students' work and attitudes (Chamberlain & Hook, 2012).
- There have been initiatives in primary school mathematics and statistics to engage schools and teachers with parents and whānau through the Home–School Partnership: Numeracy programme. An evaluation for this initiative found that the role of lead parents was a key element in the success of the project. The lead parents acted as the interface between the school and the other parents who attended. They were given some training in how to act as facilitators in this role and were introduced to the mathematical concepts being covered. The evaluation concluded that more work needed to be done to understand the training and support needs of parents in this type of bridging role, but also noted that some of them had used the experience as a stepping-stone into employment (Neill & Fisher, 2008).

As well as commentary specific to mathematics and statistics, we found one recent more general commentary about the positive contribution partnerships can make to student learning. An ERO report that discussed future pathways for secondary school students found that establishing purposeful partnerships with others in the community makes a positive contribution to student learning and development. ERO notes that such partnerships ensure the school curriculum is effective and relevant to a large majority of students, and opens up individual academic and vocational options for students (Education Review Office, 2013b).

Concluding comments

This section has canvassed a number of challenging conceptual and practical issues that confront ongoing curriculum development in the *NZC* learning area of Mathematics & Statistics. One leading international mathematics educator has recently explored issues that prevent curriculum change and improvement. He argues that changes may not be successful when inadequate attention is given to the process of developing a curriculum that is research-based (Clements, 2008). Clements acknowledges the forces that lead to a traditional curriculum approach (and there has been evidence of these in this section) and makes the case for moving towards research-based development. It is food for thought that the comparatively more rapid evolution of the Statistics strand of the curriculum has indeed been underpinned by a robust programme of research.

3. E-learning opportunities and challenges

The tensions related to the methods and purposes for learning mathematics and statistics, outlined in the previous section, can also been seen in debates on whether (and when) technologies should be used in the teaching and learning of these subjects. Some of the potential affordances of technology were alluded to in that section. The research and commentary reflect differing views on the role of technology in mathematics and statistics education, particularly in mathematics.

Many advances in real-world mathematics and statistics are based on advances in the computational power and flexibility of e-tools. In the workplace, digital technologies perform many analytical functions in both mathematics and statistics. Bootstrapping has already been mentioned in the previous section and is now included at Level 8 of the curriculum. Other examples include integration using numerical methods, and multivariate data analysis. Indeed, many types of analysis can be done with modern software that would not be possible without it.

For students, the heart of the issue lies in whether and when they too should be allowed to use technology for analytical purposes. Issues related to this question apply to the performance of investigations, and also to developing students' conceptual understanding in various areas of mathematics. As we shortly outline, the use of e-technologies to support *more traditional* mathematical learning activities does not appear to generate the same levels of debate.

Specific affordances of e-technologies

The idea of *affordances* is typically employed in research with a socio-cultural theoretical framing. Use of this term signals that learning is a complex combination of what the student knows and can already do, and the resources they can access to help them achieve their learning goals. Such resources include people and things, with e-learning tools clearly included in the latter category. 'Opportunities to learn' is a related term used when a socio-cultural framing is employed.

In the context of affordances, the availability of resources *per se* is not seen as the only thing that affects whether students do have opportunities to learn. How they (and their teachers) understand the nature of the task and their ability to achieve it, whether and how students perceive value in the learning on offer, and the potential of the support being offered all affect whether and how students actually learn.

Evidence from the literature

- Two Israeli mathematical educators have documented the affordances of a variety of technologies used for teaching algebra, and their implications for rethinking the curriculum. They address many aspects of functional thinking and show ways that technology can assist learning and understanding. They give a variety of examples, often located in more junior secondary mathematical concepts, but the principle of using technology to support the conceptual development of underlying algebraic concepts can be extended to senior secondary or tertiary mathematics (Yerushalmy & Chazan, 2008).
- Nigel Calder has written extensively about the role of digital technologies in mathematics. One book chapter discusses the ubiquitous nature of digital technology in the modern world, and how such technology offers affordances in mathematical learning trajectories across a number of different strands of the curriculum. He contrasts e-learning approaches with more traditional pedagogy, noting in particular the visual affordances of e-tools that allow students to move between multiple representations simultaneously and speedily. In this chapter, Calder also argues that e-tools give students the ability to build and interact with mathematical models dynamically. He goes on to discuss how technology can reshape learning and understanding when it is used as a means of exploring mathematical ideas. He mentions tools such as *CAS*, *Cabri-geometry, Autograph, Tinkerplots*, and *Fathom* (Calder, 2009).
- A chapter Calder co-authored explored the following themes: learning contexts and curriculum design; learners, learning and digital technology; teachers, teaching, and digital technology; and gender effect with technology use. The researchers noted that there was a paucity of research in Australasia on the most effective technology to promote deep understanding of mathematical ideas (Geiger, Forgasz, Tan, Calder, & Hill, 2012).
- The use of Tinkerplots to organise and structure data has also been discussed by other prominent authors who emphasise the affordances of digital technology, especially the dynamic options this tool opens up (see, for example, Konold & Lehrer, 2008). These researchers postulate that new technologies allow new forms of mathematics, and illustrate this through the use of Tinkerplots, which allow students to explore and view a multivariate data set in a variety of new ways using the dynamic software.
- Mathematica was utilised in an Indian classroom setting with able Year 12 students taking part in the regular curriculum, mostly using a traditional approach to calculus. The use of Mathematica supported the students to explore trigonometric integrals and Fourier transformations. Without technology, these topics would be well out of their reach. However, the visual affordances of the technology allowed students to focus on the behaviour of the graphs. Students who were involved in the programme reported that the approach had helped their understanding and confidence, and that the visualisations the software made available were a vital part of their achievement (Ghosh, 2011).
- Effective visual affordances have also been reported in two statistical enquiry research projects introduced in section 2. A focus on the behaviour of rapid assemblages of graphs can

build younger secondary students' ability to make inferences about variation in data (Pfannkuch et al., 2011). Similar types of graphical visualisations can support senior secondary students to grasp the concept of bootstrapping and its uses in statistical enquiry (Pfannkuch et al., 2013). In related papers, this research team has emphasised the use of clear design principles to ensure that such tools avoid cognitive overload and direct students' attention to salient features as the visualisation unfolds (Hipkins, 2014).

- The RAMP science review (Hipkins & Joyce, 2015) noted that simple computer simulations can build understanding of concepts and empirical relationships located at the intersection of science and mathematics. The example given in that review was that the half-life of radioactive material remains the same regardless of the volume you have to begin with. The resource developer also demonstrated the use of simulations to address simple probability calculations in genetics (Burchill, 2014).

Collectively, the affordances of e-tools, as outlined above, tend to emphasise a new means of seeing dynamic relationships between quantitative variables. Use of such e-tools to support learning can help transcend the limitations of either descriptive text or traditional computation. However, as we discuss next, e-tools can be, and are, used for a wide range of purposes, not just those at the cutting edge of conceptualisation of the mathematics curriculum.

A spectrum of uses for e-learning tools

At the more traditional end of the spectrum of curriculum thinking, simple computer programs can be used for 'skill and drill' type practice in mathematics. For example, many schools have effectively employed computer-aided instruction, such as the Khan Academy[10] or Mathletics.[11] There does not appear to be controversy about this type of use, although it is not necessarily widespread as yet.

Evidence from the literature

- A study of 193 New Zealand secondary schools explored the use of computers, calculators, and interactive whiteboards. This study found that only a third of mathematics departments had a technology policy. The researchers identified the need for professional learning with a focus on the use of technology, including pedagogical technological knowledge (PTK). They highlighted the advantages of technology, not only in investigations but also in the acquisition of skills and understanding. This study also underlined equity issues related to access to technology (Thomas et al., 2007).
- A meta-analysis of computer-aided instruction looked at the effectiveness of computer-based statistical tools, as reported in 45 studies. It concluded that computer-aided

[10] https://www.khanacademy.org/
[11] http://www.mathletics.co.nz/

instruction generally helps students to achieve better than their peers who receive face-to-face teaching. They gain extra time and can work at their own pace. Assessment embedded in the technology also contributed to their relatively stronger learning gains. Many of the programmes used took more traditional rather than exploratory approaches, but both types of approaches led to greater progress (Sosa, Berger, Saw, & Mary, 2011).

- A recent 2-year study at 20 Californian schools, and involving more than 70 teachers, carried out over the course of school year, reported that educators found the Khan Academy useful for: the modular problem sets that provide additional opportunities to practise maths skills; facilitated differentiation for students with different learning needs; and rapid feedback to support student self-directed learning. Students thought the time they spent on Khan Academy was highly positive, and their engagement levels were generally high during sessions. They perceived that the use of Khan Academy encouraged greater independence in learning (SRI Education, 2014).

E-tools for statistical investigations

The data analysis needs of statistics ensure that technology has a significant place in statistical investigations. Key resources such as Microsoft Excel are universally available and provide embedded information on effective use for learning in mathematics and statistics.

Schools make widespread use of statistical databases to support student investigations. These investigations are an essential part of course work for several internally assessed achievement standards, some of which require students to work with multivariate data bases. *CensusAtSchool*[12] provides a multivariate database of real, relevant student data gathered via a biennial online census for Year 5–13 students.

Another common site for obtaining suitable data sets is the Statistics New Zealand (SNZ) website, which provides resources for both the primary and the secondary school sector. This website created a place for SNZ to support school learning by: providing input into the development of *NZC*; providing classroom resources (see "Schools Corner"[13]); establishing an active partnership in *CensusAtSchool* (a joint project supported by SNZ, the University of Auckland, and the Ministry of Education); and providing many real data sets (via Synthetic Unit Record Files (SURFs)[14] (Forbes, 2008).

Two other statistical programmes, specifically designed to be used by secondary students to explore and analyse data sets, are *iNZight*[15] and *Genstat for Teaching and Learning*.[16] Both have been developed in New Zealand and are directly applicable to the NCEA achievement standards.

[12] Available at http://new.censusatschool.org.nz/
[13] http://www.stats.govt.nz/tools_and_services/schools_corner.aspx
[14] http://www.stats.govt.nz/tools_and_services/schools_corner/SURF%20for%20schools.aspx
[15] Available at https://www.stat.auckland.ac.nz/~wild/iNZight/
[16] Available at http://www.maths.otago.ac.nz/video/statistics/GenStatLessons/

The use of both *iNZight* and *Genstat* has been supported by widespread professional learning for secondary teachers, with large numbers attending.

E-tools for other types of investigations

Data gathering tools can be used for aspects of learning in mathematics that are not intended primarily as statistical inquiries. However, we only found one example of this type.

Evidence from the literature

- RIGEL, a mobile technology consisting of a hand-held computer, sensors and associated software, is suitable for teaching science at Years 7 and 8 and for teaching calculus at Year 13 (Fenton, 2008). RIGEL is essentially a data logging tool, but it also allows students to explore electronics and electrical circuits they have designed. An evaluation of the RIGEL project noted that using this technology increased student engagement (both physical and cognitive) and supported higher-level thinking. The technology supported authentic learning by enabling students to carry out their own research and problem solving, including Year 13 calculus activities involving problem-based learning. However, professional learning was needed to raise teacher confidence and technical competence in the technology (Fenton, 2008).

Concerns about calculators

Debate about the use of arithmetic calculators begins at the lower end of compulsory schooling, and in the secondary environment similar concerns are expressed at the use of CAS-enabled devices.[17] As the following summaries show, tensions between a more traditional approach and a future-focused approach lie behind these debates.

Evidence from the literature

- A pilot project in 22 schools, funded by the Ministry of Education, investigated the use of hand-held Computer Algebraic Systems (CAS) calculators. Over the 3 years of the project the researchers found no significant improvement in mathematical performance in the medium term (as measured by PAT:Mathematics). However, teachers and students believed the pilot led to higher levels of student understanding, which would assist progress in the senior secondary school. The research also indicated that there were significant shifts in pedagogy, with far more emphasis on rich tasks, increased interactions in the classroom (both student–student and teacher–student), and far less emphasis on textbooks or on chalk-and-talk followed by bookwork. Teachers reported that there was a significant positive effect from the professional development that taking part in the study afforded them (Neill & Maguire, 2008; Neill & Maguire, 2009).

[17] CAS stands for Computer Algebraic Systems.

- Australia has also conducted considerable research related to teaching and learning with CAS (or mathematical analysis software—MAS), as well as having CAS-enabled, high-stakes assessment in the senior secondary school. Two of the leading researchers in this area argue that curriculum developers need to draw attention to appropriate pedagogy. They also note that the functional capabilities of MAS open up opportunities for curriculum change, assessment change, and pedagogical change (Pierce & Stacey, 2010).

These examples illustrate that it is not the particular hardware nor the software that is the central issue, but the pedagogy the teacher employs when students are using e-calculators. This is important, because CAS devices have largely been superseded by apps on mobile devices and by the ready availability of software, but the pedagogical challenges of using these remain substantively the same.

A related debate concerns the uses to which e-calculators, smartphones, notebooks etc. are put. There is some controversy about removing the need for students to be able to perform the actual mathematical work themselves. However, this assumes that the purpose for using the device is to achieve the computation, whereas the pedagogy for using e-calculators stresses their potential to free up computation demands in order to allow students to focus on conceptual challenges and understanding. The use of these devices requires teachers to employ their pedagogical technological knowledge more generally, and in particular, to relate this to the specific demands of mathematics and statistics (Thomas, 2009).

Learning to work with ‘big data’ sets

Earlier sections have introduced projects that investigated effective methods of building students’ statistical reasoning skills, including the use of dynamic visualisations. The researchers who did this work had a strong focus on statistical literacy. As they explained when interviewed about the motivation for their work, such literacy is needed to equip students for participation in an evidence-based society. One of them is quoted as saying, “In a world of ‘big data’ students need to learn to use and understand the computer-driven methods of contemporary working statistics” (cited in Hipkins, 2014, p.11).

We found one international study that explored ways to support 14–16-year-old students to make inferences from big data sets using data visualisation methods. Students were asked to justify where they would most like to live based on a large and complex multivariate data set. They were able to explore that data set using a dynamic tool that enabled them to visualise the data using a variety of data displays and employing simple heuristics to reason with data in new ways (Prodromou, 2014).

Here in New Zealand the use of big data sets for teaching mathematics was the focus of one teacher's Endeavour Fellowship in the first half of 2014.[18] According to the information about this research on the Royal Society's website, this teacher-researcher (Michelle Dalrymple) has

> adapted synthesised data sets from Statistics New Zealand to make them student-friendly and these are now available on both *CensusAtSchool* and the latest version of *iNZight*. She has also compiled and re-coded a large US data set based on national health measures. This will be made available to teachers shortly. Finally, Michelle has developed two sequences of lessons based around data visualisations (for junior students) and big data (for senior students) which will be made available through *CensusAtSchool*.

Note the use of the e-resources on *CensusAtSchool* and *iNZight* to quickly share this cutting-edge work with other teachers. Educational use of big data sets appears to be an emerging area of research and commentary, where other work is likely to be in progress but is not yet published.

The relationship between mathematics and computer programming

A quite different issue relating to the use of e-tools concerns the relationship between mathematical thinking and computer programming. The focus in this sub-section is not on how e-technologies support the learning of mathematics, but rather on how the learning of mathematics might support and enhance learning related to digital technologies, and specifically computer programming.

As the British Royal Society recently noted in a report *Shut Down or Restart?: The Way Forward for Computing in UK Schools*, there is a close relationship between the logical, structured thinking of traditional mathematics and the logical, structured thinking needed to learn computer programming (Royal Society, 2012). The United Kingdom now has a new national curriculum for computing, which is replacing ICT. This includes programming as one of the aspects. The acronym for a group who has produced support materials for secondary schools, ironically, is CAS—Computing at School.[19]

Here in New Zealand the Institute of IT Professionals recently released a policy paper that proposes the creation of a whole new curriculum learning area called Digital Sciences (Institute of IT Professionals, 2014). While they endorse the recently developed NCEA achievement standards in digital technologies, they question whether the technology learning area is the best curriculum 'home' for digital sciences.

The challenge of educating all students in science as preparation for citizenship in the complex networked global society of the 21st century was discussed in the science RAMP report (Hipkins & Joyce, 2015). As already noted, similar arguments are made about statistical literacy. They are also made about computer programming. As the design and use of apps proliferates, there is an

[18] http://www.royalsociety.org.nz/teaching-learning/science-teaching-leadership-programme/profiles/2014-recipients/michelle-dalrymple/

[19] http://www.computingatschool.org.uk/data/uploads/cas_secondary.pdf

argument to be made that all students need to understand the basics of computer programming, which is a ‘new basic literacy’ for the digital age (see, for example, Jones, 2014). Given the synergies with mathematics, and the stated aim of developing strong logical thinking skills via this learning area, this is another future-focused challenge that the Ministry of Education might wish to consider as part of the overall review of the Mathematics & Statistics learning area.

E-assessment opportunities and challenges

Many of the opportunities and challenges discussed above coalesce when e-assessment is being considered. The science review noted that such assessment could simply transfer traditional assessment tasks to an online environment. Alternatively, e-assessments can be designed to test more complex capabilities that are not easily assessed on paper (Hipkins & Joyce, 2015). The following example illustrates this in the context of a discussion of STEM-related[20] mathematics learning challenges.

Evidence from the literature

- E-assessment tools can be used for formative assessments and skills-based assessments that provide real-time feedback. Such feedback can lead to better student interaction, engagement and motivation, and increases opportunities for collaboration across cultures and between locations. Real-time formative assessment provides the means for different types of activities and skills, such as problem solving and creativity (Karkkainen & Vincent-Lancrin, 2013).

An interesting issue not discussed in the science review was included in a paper presented at the Symposium on Assessment and Learner Outcomes (Victoria University of Wellington, 1–3 September 2011). This paper canvassed international experience and research evidence to show how technology can improve the validity, effectiveness, and efficiency of external assessment (Bargh, 2011). Bargh, who is employed by NZQA, developed a model of external assessment that could readily be implemented in New Zealand. He then concluded that

> In order to realise the vision of the New Zealand Curriculum students will need to learn how to discern and filter knowledge, make connections across bodies of knowledge, and create knowledge. These processes will need to be taught in class, and external assessment will need to be aligned with these objectives. (Bargh, 2011, p. 43)

There is a clear implication in this comment that curriculum integration issues will apply to e-assessment, not just to the enacted curriculum, as already discussed.

[20] Science, Technology, Engineering, Mathematics

4. Supporting students to lift achievement levels

Many students choose to drop the study of mathematics when it is no longer compulsory. This section begins with a discussion of achievement issues at lower curriculum levels that contribute to this trend. Specific issues for achievement in the senior secondary school are then discussed.

Evidence of achievement disparities for younger students

Achievement disparities start well before the senior secondary years, by which time it is likely to be too late to address them for many students.

Evidence from the literature

- In the 2012 round of the OECD Programme for International Student Assessment (PISA), average mathematics scores of New Zealand students were higher than the OECD average but had declined since 2009. They had also declined relative to other countries. Compared to earlier cycles of PISA there were larger proportions of New Zealand students with low performance in mathematics. Boys had a higher average score in 2012 than girls. The average score for Māori students was below the average score for both New Zealand students and the OECD. The average score for Pasifika students was below the average score for both New Zealand students and the OECD. The average score for low socioeconomic students declined between 2006 and 2012. Achievement in New Zealand is more closely linked to socioeconomic status than in other countries (May, 2013).
- The first National Monitoring Study (NMSSA) in Mathematics & Statistics for Year 4 and Year 8 students took place in 2013. While the average results for Year 4 students aligned with the expected level described in *NZC*, the average Year 8 results did not. Socioeconomic factors were strongly associated with performance, with students from lower-decile schools achieving lower than those who attended higher-decile schools. On average, achievement was lower for Māori and Pasifika students, and higher for Asian students, after the decile effect had been taken into account. The annual average growth between Year 4 and Year 8 for Pasifika students was lower than that for non-Pasifika students (Educational Assessment Research Unit & NZCER, in press).[21]

[21] We were granted permission to include these not-yet-released findings in the RAMP review.

Supporting learning success for younger students

Evidence from the literature

- A pilot project called Accelerating Learning in Mathematics (ALiM) targeted small groups of under-achieving primary school students within a school and aimed to accelerate their learning with targeted teaching for a short period of time. Evaluation of ALiM demonstrated that the approach had been highly successful (Neill, Fisher, & Dingle, 2010). This programme has been scaled up with continuing success, and there is supporting information online.[22]
- An ERO study looked at what primary schools are doing to raise the mathematics achievement of students in Years 4 to 8, including the extent to which they are engaging in accelerating progress for priority learners (Education Review Office, 2013a). ERO found that achievement information is widely collected, but that 50 percent of schools do not use the information to change their taught curriculum.

Challenges for lifting achievement in senior maths and stats

Even when they have managed to stay on a mathematics learning pathway into their senior secondary school years, there is evidence that some groups of students need greater support in mathematics and statistics than they are currently getting. This issue is concerning because mathematics is one of the so-called STEM subjects, and successfully gaining NCEA achievement standards in this learning area opens up many career pathways.

Evidence from the literature

- Buntting, Jones, McKinley and Gan (2013) note that "Māori student exclusion from STEM subjects is laid down very early in New Zealand's secondary education system. While Māori students enter secondary school with very similar backgrounds in STEM subjects to their Asian, Pākehā and Pasifika student counterparts they are quickly excluded from gaining higher level NCEA qualifications in these subject areas" (p. 32).
- The comprehensive STEM review undertaken by Buntting et al. (2013) also highlights that New Zealand students value mathematics at close to the international average. However, their confidence in learning mathematics and their attitudes towards learning it are relatively low.
- An international study developed summary profiles of upper secondary mathematics education in 24 nations. These profiles included: overall participation rates; patterns of participation; and the content and level of provision, with particular reference to the definition of general mathematics and the specific pathways it opens up. The New Zealand profile states that over 95 percent of students enrolled in Level 1 are enrolled for mathematics or statistics

[22] http://nzmaths.co.nz/accelerating-learning

standards, and this drops to just above 80 percent at Year 12 (Hodgen, Pepper, Sturman, & Ruddock, 2010).

- A study that investigated the relationship between the complexity of University Entrance (UE) criteria and NCEA, involving students who had taken at least one Level 3 standard, concluded that the major choke-point for transition to further study was attaining at least 14 credits in university-approved subjects. The more university-approved subjects students had obtained, the more likely students were to gain UE, with 87 percent of students of those with five approved subjects gaining UE. However, Māori and Pasifika students were more likely to limit their chances at gaining UE because they were more likely to be taking only two to three approved subjects (Smith & Timperley, 2008). This example is not specific to mathematics, but illustrates the multi-faceted nature of the challenges facing priority learners.
- Student achievement below expected levels is not confined to schools in poor socioeconomic areas. However, a combination of school decile and *middle leadership practices* is a good overall predictor of student academic achievement at Levels 2 and 3 NCEA (Highfield, 2012).

Linguistic demands of mathematics and statistics

As we also noted in the RAMP science report (Hipkins & Joyce, 2015), mathematics and statistics make specific types of communication demands that can affect achievement, particularly for priority learners. Mathematics can be thought of as entailing the learning of a specific "language of numbers" (a concept that makes strong links to the *NZC* key competency of *using language, symbols and texts*).[23] However, precise and accurate use of *everyday* language can also be a challenge, as the following papers discuss.

Evidence from the literature

- Some externally assessed standards make complex literacy demands, with Māori and Pasifika learners less likely to enrol in these standards and less likely to pass if they do (Wilson & McNaughton, 2014). For example, while about 70 percent of Pākehā who entered externally assessed calculus achievement standards achieved them, just 40 percent of Pasifika, and 50 percent of Māori achieved these standards (Buntting et al., 2013).
- While mathematical thinking can, in principle, be communicated in formalised language, to attain Merit or Excellence in NCEA students also need to be able to justify their choices and show insight. Doing so requires literacy skills in natural language. Thus the demonstration of mathematical argumentation requires fluency in both everyday written skills and mathematical discourse (Boero, Douek, & Ferrari, 2008).
- Statistics, in particular, makes strong linguistic demands, because students must engage with the language requirements of the real-life contexts they are exploring, and then communicate

[23] It is interesting that none of the papers we found focus on this type of mathematic language *as such*.

their conclusions using significant amounts of natural language (see, for example, Arnold, et al., 2011; Pfannkuch & Ben-Zvi, 2011).

- One study investigated the extent of the disadvantages faced in senior secondary school mathematics classes by students who have English as an additional language (EAL). The study explored specific language features that cause difficulty. These include: understanding technical language and its role in communication; student self-awareness of language difficulties; and practical steps to ameliorate language effects. The study recommended relevant professional development for teachers in various supports that need to be provided for EAL students, including better understanding of these students' language and mathematics proficiency at the time they enter New Zealand classrooms, and the development of special courses in English mathematical discourse (Neville-Barton & Barton, 2005).

Lifting achievement levels

Some research teams have investigated the consequences of providing specific types of support for students as they prepare for NCEA assessments in mathematics and statistics. These can be broadly grouped into support that focuses on identifying and proactively addressing known *learning challenges* and known *motivation* issues.

Evidence from the literature

- A deliberate and planned literacy intervention has the potential to lead to an increase in the numbers of Excellence and Merit passes. The Ministry of Education website, Te Kete Ipurangi (TKI), gives some background to support teachers to address linguistic demands in mathematics and statistics.[24] The information addresses the literacy demands of various parts of an inquiry cycle, includes advice about general literacy challenges for priority learners, and provides specific information about the literacy demands of mathematics and statistics. In view of the issues outlined above, the support given here is clearly highly relevant.
- Where specific achievement standards have been identified as problematic because they make particular types of literacy demands, working with teachers to highlight achievement patterns motivates them to proactively address the literacy challenges implied. This can lead to increases in achievement levels (Wilson & McNaughton, 2014).
- A study of the influence of the Secondary Numeracy Project (SNP) on the senior secondary school explored how SNP approaches were used in Year 11 classrooms in successful numeracy schools, and how this use affected practice. The researchers linked the use of SNP approaches to both unit standards and achievement standards, concluding that SNP practices have a place in the senior school. This work is particularly pertinent to the numeracy unit standards, which provide a pathway for achieving the numeracy credits needed to gain Level

[24] http://literacyonline.tki.org.nz/Literacy-Online/Secondary-Literacy/Teacher-needs/Literacy-in-the-learning-areas/Literacy-in-Mathematics

1 of NCEA.[25] However, number deficiencies can also inhibit progress in further mathematics, so the issue might also be pertinent for more academic students (Harvey & Averill, 2009).

- One study explored students' and teachers' perceptions of in-class formative assessment practices. Teachers saw oral feedback as the most effective and efficient method, whereas students showed a preference for written feedback they could then engage with during peer-to-peer interactions. The paper also argues that the collective development of, and engagement with, formative assessment practices has the potential to enhance both individual and collective learning within the normal classroom setting (Rawlins, 2010).
- Continuing research seeks to examine how teachers adapt their teaching practices in light of the requirements of high-stakes assessment. Rawlins and Anthony are currently exploring the extent to which course design, content coverage, teaching resources, and pedagogies are affected by the presence of high-stakes assessment, and the effect of these assessment systems on assessment-for-learning practices.[26]

We now turn our attention to questions of motivation and engagement. Although motivation and engagement will obviously be interconnected with achievement challenges, the papers we found raise additional issues that could be useful to consider.

Evidence from the literature

- A report on the age-16 phase of the longitudinal Competent Children, Competent Learners study noted that science and mathematics teachers were less likely than teachers of other subjects to identify high levels of student engagement or motivation in mathematics (Wylie, Hipkins, & Hodgen, 2009). The young people in this study (around 500) were more likely to identify science and mathematics as least favourite subjects compared to others they were taking.
- A recent TLRI-funded study explored factors that have an impact on the decision to transition from secondary to tertiary studies in mathematics, including the influence of student motivation. The researchers noted that "Teachers and lecturers are more likely to think that calculus is of benefit to society than students do, and are also happier with the amount of calculus in their course than students are. Students have reservations about the study of calculus that do not align with their personal life ambitions" (Thomas et al., 2010, p. 3). There is a clear implication here that students need support to see the relevance and usefulness of advanced abstract mathematical conceptual thinking if they are going to consider furthering their mathematical education at the tertiary level.
- A study of Year 12 and 13 students explored the positive impact that teachers have on motivation and achievement when they show respect for students. Respect can be shown

[25] http://www.nzqa.govt.nz/qualifications-standards/qualifications/ncea/subjects/literacy-and-numeracy/literacy-and-numeracy-unit-standards/

[26] http://www.massey.ac.nz/massey/fms/Colleges/Institute%20of%20Education/Documents/2013/Joint%20Staff%20Postgraduate%20Student%20Research%20Projects%202014.pdf

through: the congruence of talk and action; teacher professionalism (well organised and prepared, and holding high expectations); a one-to-one learning focus that includes feedback and feed-forward; a willingness to listen to students; and providing opportunities for students to develop their own strategies, work at their own pace, and have their errors treated constructively (Averill & Clark, 2012).

- One recent large-scale Australian study of students' STEM pathways choices noted interesting differences between the discussion of engagement issues in mathematics and in the sciences. The reviewers noted that discussion of engagement issues in the sciences focuses on the importance of context and personal relevance, along with critique of the restricted range of pedagogies that science teachers tend to employ. However, engagement issues in mathematics typically emphasise "intellectual and affective engagement accompanying conceptual learning, providing tasks that address individual differences and use authentic contexts, and the importance of resilience and self efficacy in taking risks associated with problem-solving activity" (Tytler et al., 2008, p.116). These issues for engagement in mathematics relate predominantly to the type of complex and challenging learning experiences that we noted in section 2 as requiring a paradigm shift from traditional mathematics learning. One clear implication is that this type of shift creates engagement challenges of its own, and it should not be assumed that it will automatically solve engagement challenges associated with formal, traditional learning.
- Meyer et. al.'s longitudinal study of student motivation and NCEA was discussed in considerable detail in the RAMP science report (Hipkins & Joyce, 2015). We revisited this work to check for any specific references to motivation in mathematics. Looking at the relationship between motivation and NCEA results, the researchers concluded that: Pacific students rated both family and friend influences as more important to both their best and worst marks than did European, Māori, and Asian students; more Māori and Pacific students attributed their best marks less to ability and effort than did European and Asian students; and more Māori and Pacific parents were positive about what they saw as increased motivation and opportunities for young people to achieve with NCEA in comparison with the previous system (Meyer, McClure, Walkey, Weir, & McKenzie, 2009).
- One important generic finding from Meyer et al.'s study was that teachers were inclined to view motivation and ability as fixed, rather than a dynamic orientation that can be changed. Teachers at higher-decile schools were less likely overall to see motivation as dynamic, while those in wharekura and low-decile schools were more likely to believe that all students can be motivated to do their best (Meyer, Weir, McClure, Walkey, & McKenzie, 2007).

Lifting achievement across the curriculum (with implications for maths and stats)

A recent ERO report investigated the way in which 40 schools analysed and responded to their NCEA reports. It identified characteristics of schools that had effectively raised standards. These included high levels of coordination and organisation with a strong focus on improvement, as well as strategies aimed at individual students. The report contrasted features of schools that had made some good progress, and schools not making a difference to overall patterns of student achievement. It recommended that schools include a focus on tracking and supporting individual students, and that they should implement improved curriculum structures, options, and delivery (Education Review Office, 2014a).

Alignment of curriculum and assessment

A specific challenge for addressing the engagement and achievement of struggling learners is that, while there might be clear indications of the sorts of changes needed, students, teachers, and parents are all influenced by success in high-stakes assessments. In other words, what is currently assessed sends strong signals about what is valued. The alignment challenge implied here is not unique to New Zealand.

Evidence from the literature

- Proficiency in mathematics entails more than just a demonstration of knowledge. Students need to be able to enact what they know through the effective use of strategies and to display their metacognition. These broader aims have implications for what to measure, how to measure, and what should be valued for mathematical education and for society (Schoenfield, 2007). In this book chapter, Schoenfield acknowledges that assessment needs to meet the needs of the many different stakeholders, from students through to policy makers, but warns against distorting the curriculum and stifling innovation via what is assessed. While many of the examples he uses are drawn from primary or junior secondary mathematics, the principles are readily transferable to older assessment cohorts.
- Hong Kong has recently undertaken a curriculum alignment exercise to see how the intentions of its new senior secondary curriculum are reflected in the public examination of the curriculum. The overall curriculum aims include: developing critical and creative thinking; mathematical inquiry and reasoning; and the use of mathematics to formulate and solve problems in both mathematical and daily life contexts. The alignment review found that these aims are not adequately represented in the only available public examination. The reviewers suggest there is a need to create a more appropriate alignment between the curriculum and the assessment methods, including incorporating school-based assessment (Leung, Leung, & Zuo, 2013). Hong Kong is well known as a highly successful nation for performance in international PISA assessments. Yet clearly at least some curriculum officials

are not satisfied that their own national assessment has a focus on what matters for students' ongoing mathematical development.

- A comparison of STEM initiatives in seven different education systems concluded that engaging or re-engaging children and young people in science and mathematics in ways that are authentic, interesting, and meaningful to them must begin with assessment. The researchers place a strong emphasis on formative assessment, but also stress that the way that a curriculum is measured influences the outcomes on student learning. They note that "assessment practice needs to be shaped to support these curriculum goals and developments" (Howes, Kaneva, Swanson, & Williams, 2013, p. 16).
- Closer to home, the Australian STEM review of factors that have an impact on the primary–secondary transition also commented on the negative effects of summative assessment that focuses on "low level content and fails to support the type of challenge and interest that engages students in thinking mathematically" (Tytler et al., 2008, p. 118). This report does not focus on the senior secondary school, but the challenge identified is likely to exist across all the years of schooling.
- The influence of NCEA on teachers' curriculum thinking (across all learning areas) is well documented from regular national surveys of New Zealand's secondary teachers (see, for example, Hipkins, 2013). The most recent national survey (in 2012) found that teachers of mathematics and statistics, and the sciences, were more likely than teachers in other learning areas to perceive that the recent alignment exercise in *NZC* had not resulted in the production of achievement standards that reflect the intent of *NZC* in their learning area. It is not possible to tell from the data whether teachers who disagreed thought the achievement standards were too traditional in their focus. It is quite possible that some thought the new focus on statistical reasoning has 'gone too far' in making changes. Either way, alignment challenges are alive and well, both nationally and internationally.
- The e-learning fellowship research project on using RIGEL mobile sensor technology, cited earlier, argues for making changes to NCEA so that summative assessment tasks better align with constructivist teaching pedagogies that encourage exploration rather than the mere following or recall of procedures (Fenton, 2008).
- The NCEA Level 1 numeracy requirements can be met by using either specified achievement standards or a package of numeracy unit standards.[27] The latter have an emphasis on using number, measurement, and statistics. In particular, students need basic statistical literacy to be able to critically respond to the many statistical claims they encounter. This relates to the need to estimate the reasonableness and the precision of results. However, Neill (2005) points out that while estimation is the second most commonly used mathematical skill in the workplace, there is evidence that it is under-represented in school mathematics. This is a case

[27] http://www.nzqa.govt.nz/qualifications-standards/qualifications/ncea/subjects/literacy-and-numeracy/literacy-and-numeracy-unit-standards/clarifications/

where an aspect of assessment using unit standards does align with the high-level intent of *NZC* but is not widely taken up because of the mode of assessment.

Culturally responsive pedagogy

Interestingly, we found a number of New Zealand-based research projects and commentaries that have addressed issues relating to culture, ethno-mathematics, and equity in mathematics. There appears to be rather more research related to what might be broadly termed ‘culturally responsive pedagogy’ for mathematics than we found for the Science learning area. In the science review, the main focus for the commentary we found concerned the need for teachers to see it as their explicit responsibility to *translate* traditional science content into learning experiences that help students make explicit links between discipline-based conceptual learning and stories or contexts that can allow students to talk their knowledge into place (Hipkins & Joyce, 2015). At least within the research community, some of that translation appears to be already actively taking place in statistics and mathematics, and some of this work has stretched back several decades, beginning with the pioneering work of Bill Barton, Megan Clark and Sharleen Forbes. However, these research endeavours have not always readily transferred to practice. The proposed shifts in pedagogy are not without challenges, as the following papers make clear.

Evidence from the literature

- In the eyes of primary school students in Māori-medium schools, teachers should be ‘providers’ of mathematics learning (Hawera, Taylor, & Herewini, 2009). These authors contrast such student expectations with the teacher’s role as the orchestrator of more active student participation in their learning, including exploring alternative mathematical strategies to those suggested by the teacher, and justifying their own mathematical ideas. While this study is located in the primary setting, the tension in expectations outlined could apply equally well to secondary classrooms.
- One recent study involving Year 10 students from three mid- to low-decile schools found that teachers were more likely to respond to cultural issues via their classroom interactions than through the choice to locate mathematical tasks in the heritage and cultures of Māori and Pacific people. The researcher found no specific examples of mathematics learning tasks drawn from Māori culture, and across the curriculum Mathematics & Statistics was the learning area with the highest number of observed lessons that made no reference to Māori cultural knowledge or language (Averill, 2012).
- An inquiry-based pedagogy that draws on the cultural capital of the Pasifika students, and that was located within core Pasifika beliefs of reciprocity, collectivism, and communalism demonstrated that this type of pedagogy can increase levels of engagement and positive attitudes towards mathematics. This study was set in a low-decile primary classroom and reported that the students constructed views of themselves as competent learners, both

individually and collectively, when this pedagogy was successfully employed (Hunter & Anthony, 2011).

The following more generic ERO studies add to the overall picture of the challenge of addressing culturally responsive pedagogy, and the potential to lift achievement for priority learners by doing so.

Evidence from the literature

- ERO has produced a report investigating how 302 schools, including 40 secondary schools, engage with Pacific learners and act to improve their educational outcomes. It noted that these students constitute a non-homogeneous group. Nonetheless, leadership (including setting high expectations), partnership, utilising assessment data, and a relevant and responsive curriculum are all important factors. While not mathematics-specific, there are resonances with the research outlined above (Education Review Office, 2012).
- A report from 74 secondary schools identified features exhibited by the 10 most effective schools in preparing students for future pathways. These included having: a school curriculum that was effective for a large majority of the students enrolled at the school; an extensive range of vocational and academic options; processes and practices that encouraged the individualisation of student pathways; individual courses and school-wide initiatives that encouraged students to develop leadership and self-management skills; some effective initiatives for Māori and Pasifika students; and some effective self-review systems (Education Review Office, 2013b).
- In a very recent report, ERO identify good practice in seven secondary schools with a decile rating of 5 or below, and with low disciplinary statistics, and whose Level 2 NCEA data have shown good achievement levels. The report identifies good practice within school culture, students' learning, and leadership. It also gives practical advice on how to work towards equitable outcomes (Education Review Office, 2014b). The message here is that low-decile schools can work to transcend learning challenges, and some are already successfully doing so.
- Meyer et al.'s longitudinal study of motivation and NCEA, already cited, emphasises the importance of teachers who hold high expectations and support students to meet learning challenges. Both Māori and Pasifika students and parents/whānau favoured Merit and Excellence certificate endorsements because these motivated students to work harder (Graham, Meyer, McKenzie, McClure, & Weir, 2010).
- The individualism promoted by NCEA can be seen as a pressure on indigenous students. An alternative view is that success in NCEA does not depend on how others perform, so it may be suited to encouraging a *collective* approach to achieving excellence, as appropriate to Māori world views (Graham et al., 2010).

Post-school transitions

We found a number of papers, many of them generated by the same research team, that discuss issues and challenges for keeping students learning mathematics in the years beyond school. Most of these papers explore the secondary–tertiary transition.

Evidence from the literature

- One study focused on the last 2 years of secondary mathematics and the first 2 years of tertiary mathematics study. Whereas the number of students taking mathematics at Year 13 has remained reasonably constant since the mid-1990s, the number of graduates majoring in the mathematical sciences at the University of Auckland has risen steadily over that same period. The researchers investigated the interface between the *NZC* aims and objectives and the needs of students to undertake university mathematics or statistics. With the aim of increasing the number of students taking advanced mathematics courses at Year 12 and 13, the researchers argue for an emerging curriculum vision that is broad, contemporary, active, and coherent. They note that senior secondary mathematics has been slow to change in New Zealand, with the exception of the statistics strand (Barton, Clark, & Sheryn, 2010).
- Prior academic achievement at school is the strongest predictor of university performance, especially in the first year of tertiary study, irrespective of what subjects are studied at school. Factors that influence university performance include motivation, self-discipline, confidence, study habits, time management skills, family/peer support, attending an institution of choice, and studying preferred courses or subjects. These factors are independent of what is being studied, and these traits will stand students in good stead at university. The author contends that NCEA Level 3 could be regarded as a proxy for some or all of the factors listed above. An implication for universities is that they should perhaps re-examine their admissions criteria to focus on overall achievement rather than requiring achievement in a particular school subject (Engler, 2010).
- A survey exploring key differences between sectors was sent to all secondary schools and to 31 tertiary institutions, including polytechnics, universities, wānanga, and institutes of technology, aimed at teachers or lecturers who teach calculus and students who learn it. Thirty-five secondary students and 42 first-year tertiary students were interviewed about their learning experiences. The researchers found that tertiary students tended to be less confident, and their motivation had shifted from being largely intrinsic while at school towards being more extrinsic at the tertiary level. The researchers identified a need to explore how to create better alignment between the New Zealand secondary Mathematics & Statistics curriculum and stage 1 courses at universities, noting that teachers and lecturers lack knowledge and awareness of the other sector (Thomas et al., 2010).
- Another paper arising from this research showed that there were different emphases on what is valued in each sector. The researchers highlighted similarities and differences

between the two sectors, and argued that there is a clear lack of understanding of the issues involved in the transition from the other group's perspective. The secondary teachers were reported as being more likely to perceive that there is more teaching to the assessment in senior secondary school since the implementation of NCEA (Hong et al., 2009).

- One study analysed the achievement of incoming tertiary students in order to explore the relationship between the final secondary school qualifications in Mathematics with Calculus and results from the core first-year mathematics papers at Canterbury University. Cohorts since 2005 were included, when students enrolled for Level 3 NCEA qualifications for the first time. This study found that NCEA results were a strong predictor of success in tertiary mathematics, though the researchers pointed out that occasionally individuals go against this trend. Overall, their findings indicated that NCEA in mathematics formed a solid foundation for tertiary mathematics study (James, Montelle, & Williams, 2008).
- Statistics New Zealand has been involved in a range of initiatives aimed at up-skilling the statistical capability of a wide range of people, firstly within Statistics New Zealand, but extending also to state sector agencies, and people in community groups in the general public. These initiatives have helped to increase the "general capability in the community to use statistics, resulting in better decision making, to maintain our own capacity to produce high quality official statistics and to ensure government officials provide advice to government that is based on sound statistical analysis" (Forbes, 2008, p. 1).

5. Areas where teachers need more support

Evidence of the need for support

A clear theme in the research literature is the *complexity* of the curriculum and assessment thinking now being asked of teachers. The challenges they experience imply a need for ongoing professional learning and support. While the predominant focus of the science review (Hipkins & Joyce, 2015) was on pedagogical support as teachers try out new ideas and make changes in their practice, the mathematics commentary that we found is directed more at teachers' own knowledge and strength of background in the discipline area.

Evidence from the literature

- A comparative study of New Zealand, Australia and the United Kingdom discussed the mathematical needs of secondary teachers, in particular senior secondary teachers who are preparing students for tertiary mathematical study. Despite an increasing need for both deeper and wider mathematical understanding, the levels of mathematical qualifications of secondary teachers in all three countries appear to have declined since the 1970s, with the current proportion of teachers without post-secondary mathematics qualifications being between 20 and 30 percent. The implications for mathematics learning are discussed, and it is recommended that all secondary mathematics teachers have the equivalent of second-year university mathematics (Barton & Sheryn, 2009).
- The Australian STEM review cited in earlier sections notes that the values and aesthetics of STEM subjects pose significant challenges for teachers of mathematics and sciences who are not trained in these subjects. Effective teaching that captures students' interest, and helps make strong connections between the intended concepts/content and their lives, presumes "substantial teacher knowledge and commitment to both the subject matter and the way it is appreciated and learnt" (Tytler et al., 2008, p. 117).
- Teachers often lack a strong statistical background. To effectively support the development of students' statistical reasoning capabilities, many teachers need to develop their own ability to: understand key statistical concepts; explore and learn from data; argue statistically; use formative assessment; and learn to understand students' reasoning (Pfannkuch & Ben-Zvi, 2011).
- An analysis of the variance in 2007 NCEA results in English, mathematics and science was carried out in 41 Auckland secondary schools. This study found greater within-school variance between the three departments in the school (when compared with national data for their subject/decile) than across the deciles. Some departments appeared to provide very

good support for achievement, while others provided poor support. The study concludes that attention needs to be given to departmental leadership of teaching and learning (Highfield, 2010).

- In the NZCER 2012 National Survey, maths and science teachers were more likely than teachers of other subjects to disagree that feedback from NZQA had helped clarify the intent of the new achievement standards, or that national moderators' reports were helpful. They were more likely to strongly agree that they had sent optional teacher-selected evidence for moderation. They were more likely to be unsure or to disagree that best practice workshops gave them more confidence to answer students' assessment questions, that they had gained a better understanding of how to make holistic judgements, or that they now have more achievement-focused conversations with students (Hipkins, 2013).
- Another generic study looked at the influence of professional development as an agent of change in a new environment of the standard-based NCEA assessment system. It revealed a positive relationship between professional satisfaction and teacher involvement in setting priorities for professional development. Other positive features were networking, personalised learning, and facilitator expertise (Starkey et al., 2009).
- In the NZCER 2009 National Survey of secondary schools, science and mathematics teachers were less likely than teachers of other subjects to see pedagogies that integrate the key competencies into the curriculum as important, or to say that students could often experience these types of learning in their classes. They were less likely to say that exploring the vision and values part of *NZ*C would be important to implementation. However, taking part in whole-school exploration of *NZC*, as well as within-department exploration, was not common across secondary schools in general. The report suggests that the opportunity could have been missed to explore beliefs shared within subject-based teams (e.g., about the purposes for learning the subject) in the wider school context and from the high-level perspective of *NZC*'s future-focused front section (Hipkins, 2010).

Interesting assessment-related opportunities and challenges

A policy document, *Directions for Assessment in New Zealand*, produced at the request of the Ministry of Education, recommended that all young people be educated in ways that develop their capacity to assess their own learning. Doing so requires building the "assessment capability" of both students and teachers, and of others in the wider system such as school leaders, Ministry personnel, parents, boards of trustees, and teacher educators (Absalom, Flockton, Hattie, Hipkins, & Reid, 2009). As with the various 'literacies' discussed earlier in the review, strengthening teachers' abilities to work with aspects of number and statistics will need to be an integral part of any professional learning initiatives that address this challenge. While there are many other dimensions to the professional knowledge and skills needed by assessment-capable teachers, this mathematical dimension might offer an interesting opportunity to ‘double dip’ by aligning the intended learning with the paradigm shift in mathematics discussed in section 2.

In the science report we noted the complexity required of teachers’ curriculum/assessment thinking and decision making in the senior secondary school when working with NCEA. We further noted national survey findings that suggest science and mathematics teachers may need additional professional learning support, particularly when achievement standards are revised to better align assessment with the curriculum (Hipkins & Joyce, 2015). In the light of these challenges, it is interesting that a recent Australian study identified four conditions needed if assessment standards are to be used in ways that inform teaching and learning to improve both the curriculum and assessment. These conditions are that:

- the purposes and functions of the standards are clear
- the representations in the standards are understood by teachers
- moderation practice is a co-construction involving “consensus moderation” actively involving teachers
- the assessment community is actively involved.

The researchers note that standards can serve to inform teachers about curricular intent and the demands of assessment tasks relative to that intent (Klenowski & Wyatt-Smith, 2010). The challenge implied here is how we might more effectively leverage ongoing moderation and other work that strengthens teachers’ assessment decision making in ways that simultaneously strengthen their curriculum knowledge and teaching skills.

‘Teaching as inquiry’ as professional learning

A research study of 27 teachers at decile 1 and 2 schools in South Auckland investigated the effectiveness of teacher inquiry as a professional development strategy for senior mathematics teachers. The researchers found that a teaching inquiry can constitute effective professional learning. However, caveats include that the focus of the research needs to be determined by the teacher and the research needs to be undertaken at a level that is proportionate to the experience and intentions of the teacher (Bartholomew & Barton, 2007).

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Appendix: Methodology for searching and constructing the Endnote file

To support the Review and Maintenance Programme (RAMP) for the Mathematics & Statistics learning area, we undertook to gather evidence about current curriculum content, pedagogical and assessment practices and student achievement in the context of the learning area in *The New Zealand Curriculum* (*NZC*) and NCEA at Levels 6–8. We used the literature we found to build an Endnote file that provides summaries of the research papers, plus commentaries, and other relevant documents we found.

Search parameters

We gave priority to papers published since around 2010 (when the *NZC*/NCEA alignment was being undertaken) unless we judged a piece of work completed before then to be substantive and relevant to matters raised in the more recent papers.

We mainly focused on New Zealand-based publications with an explicit focus on NCEA and/or *NZC* and learning mathematics and statistics in the senior secondary school years. However, seminal international research and summaries were included where they addressed significant gaps, had something of real interest to say in terms of the review questions, or represented significant meta-analyses of a large body of relevant international research.

We began our search with a range of sources that included Google Scholar, the New Zealand Educational Theses Database, the New Zealand Council for Educational Research's research papers, Ministry of Education research reports, the *Handbook of International Research in Mathematics Education* , Second Edition, 2012), and various journals for mathematics and statistics researchers and teachers. Where we were aware of interesting in-progress or unpublished research, particularly in schools, we contacted researchers, educators and/or teachers directly to check for papers written or presentations given.

Two particularly rich sources of references were:

Teaching secondary school mathematics and statistics: Evidence-based practice, Vol. 1 and 2 (Averill & Harvey, 2009). Many of the authors of chapters in this book are also referenced in the literature Endnote file.

Effective pedagogy in mathematics/Pāngarau: Best evidence synthesis iteration [BES] (Anthony & Walshaw, 2007) and its associated summary, *Effective pedagogy in mathematics* (Anthony & Walshaw, 2009).

Search terms and keywords for the Endnote file

As well as using key search terms to locate papers, we needed to build a typology of key words to enter into the Endnote file so that it can function as a searchable data base. First-tier search terms were derived from the description of the overall RAMP review. Some second-tier search terms were added to cover issues we predicted might arise, based on our understanding of the future-focused literature, and from our knowledge of NZCER's ongoing programme of research on *NZC* and NCEA.

1. *NZC* alignment:

- teachers' professional learning
- curriculum integration
- dispositions (motivation, engagement, agency)

2. Innovative programmes:

- non-traditional outcomes (Nature of Science, action competence, inquiry competencies, literacies, etc.)
- future focused
- technology (online teaching)

3. NCEA:

- assessment
- online assessment

4. Priority learners:

- Māori, Pasifika, special learning needs
- equity
- diversity

5. Pathways:

- vocational
- STEM

Three other keywords further categorise collected sources as:

- research;
- evaluation; or
- commentary.

For each entry a short summary of the research, evaluation or commentary was entered in the Notes section. The summaries focus on areas of interest for the RAMP review, so they do not necessarily include all of the findings or recommendations from a particular piece of work.